Classical Relativistic Hadrodynamics for Extended Nucleons

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ABSTRACT

Classical relativistic hadrodynamics provides a natural covariant microscopic approach to relativistic heavy-ion collisions.

With minimal input, this approach leads to an inherent nonlocality in spacetime and allows for the treatment of nonequilibrium effects.

Our goal is an exact calculation of ultrarelativistic heavy-ion collisions on the basis of conventional nuclear physics, taking into account hadronic degrees of freedom only, via meson fields.

OUTLINE

- review theoretical progress
  - classical relativistic point-particle equations of motion including self-interaction
  - generalization for finite nucleon size
- present preliminary nucleon-nucleon scattering results for p_{lab}/A = 14.6 GeV/c and 200 GeV/c
- discuss future calculations and open questions

INTRODUCTION

Classical relativistic hadrodynamics provides a natural covariant microscopic approach to relativistic nucleus-nucleus collisions that includes automatically

- spacetime nonlocality and retardation

- nonequilibrium phenomena

- simultaneous interaction among all nucleons

- particle production

MOTIVATION

- at AGS, CERN, and RHIC energies:
  - interaction time extremely short
  - nucleon mean free path, force range, size, and internucleon separation comparable
  - Lorentz contraction factor γ possibly huge
- de Broglie wavelength of projectile nucleons very small
- manifestly Lorentz-covariant microscopic many-body approach is necessary

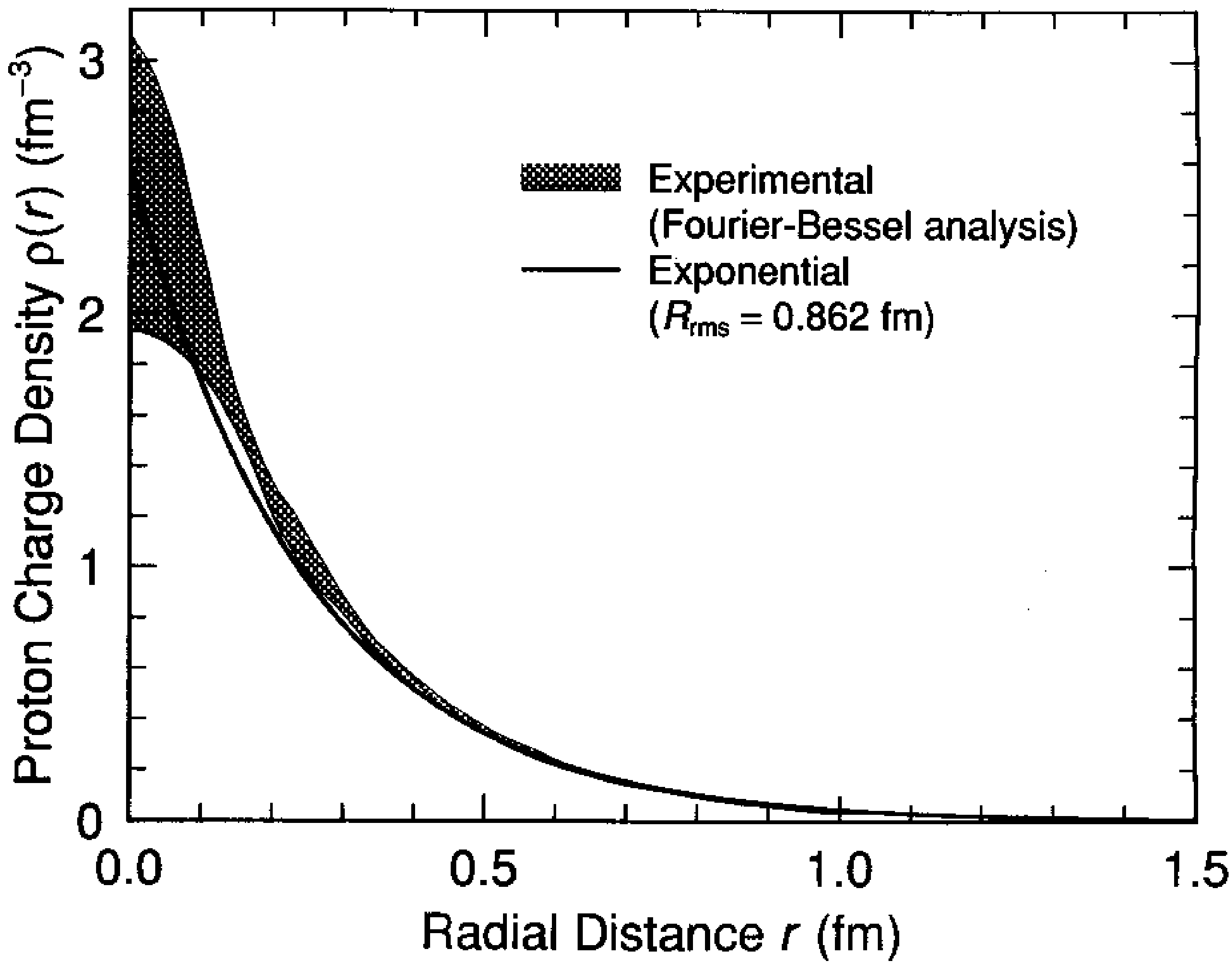
FEATURES

- physics assumptions
  - energy/momentum conservation
  - Lorentz covariance
  - non-point (rigid) nucleons interact through meson exchange
  - classical approximation
  - minimal coupling between particles and fields
- exact solution attempted
  - no mean-field approximation
  - no expansion in coupling strength (perturbation theory)

INPUT CONSTANTS

- masses
  - nucleon: $M = \frac{1}{2}(M_{\mathrm{p}} + M_{\mathrm{n}}) = 938.92\ \mathrm{MeV}$
  - meson fields: $m_{\mathrm{s}} = m_\sigma = 550\ \mathrm{MeV}$, $m_{\mathrm{v}} = m_\omega = 781.95\ \mathrm{MeV}$
- exponential nucleon charge distribution: $R_{\mathrm{rms}} = 0.862\ \mathrm{fm}$
- coupling constants
  - Serot-Walecka: $g_{\mathrm{s}}^2 = 7.29$, $g_{\mathrm{v}}^2 = 10.81$
  - Bryan-Scott: $g_{\mathrm{s}}^2 = 8.19$, $g_{\mathrm{v}}^2 = 17.26$

Exponential Proton Charge Density



ACTION

- action

$$I = \overbrace{-\sum_{i=1}^{N}\int d\tau_i \frac{M_0}{2}\dot{q}_i^2}^{\text{nucleons}} + \frac{1}{8\pi}\overbrace{\int d^4x\left((\partial\phi)^2 - m_{\mathrm{s}}^2\phi^2\right)}^{\text{scalar field}}$$

$$- \frac{1}{8\pi}\underbrace{\int d^4x\left(\frac{1}{2}F^2 - m_{\mathrm{v}}^2 A^2\right)}_{\text{vector field}} - \underbrace{\int d^4x\,(j\phi + J\cdot A)}_{\text{interaction}}$$

where $F^{\mu\nu} = \partial^{[\mu}A^{\nu]} = \partial^\mu A^\nu - \partial^\nu A^\mu$

- sources (point particles)
  - scalar: $j(x) = g_{\mathrm{s}}\sum_i \int d\tau_i \delta^{(4)}(x - q_i)\sqrt{\dot{q}_i^2}$
  - vector: $J^\mu(x) = g_{\mathrm{v}}\sum_i \int d\tau_i \delta^{(4)}(x - q_i)\dot{q}_i^\mu$

VARIATION OF ACTION

- variation of action δI with respect to $\delta q_i^\mu(\tau_i)$, $\delta\phi(x)$, $\delta A^\mu(x)$ yields

$$\begin{aligned}
\left(\partial^2 + m_{\mathrm{S}}^2\right)\phi &= -4\pi j \\
\left(\partial^2 + m_{\mathrm{V}}^2\right)A^\mu &= 4\pi J^\mu \\
(M_0 + g_{\mathrm{S}}\phi)a_i^\mu &= g_{\mathrm{S}}\mathcal{P}_i^{\mu\nu}\partial_\nu\phi_i + g_{\mathrm{V}}F_i{}^\mu{}_\nu v_i^\nu
\end{aligned}$$

  where $v_i = \dot{q}_i$, $a = \ddot{q}_i$, $\mathcal{P}_i^{\mu\nu} = g^{\mu\nu} - v_i^\mu v_i^\nu$, $\phi_i = \phi(q_i)$, $F_i = F(q_i)$

- formal solution via Klein-Gordon and Proca equation Green functions $G_{\mathrm{S}}^{\mu\nu}(x,x')$ and $G_{\mathrm{V}}^{\mu\nu}(x,x')$

GREEN FUNCTIONS

- Green function definition

$$\left(\partial^2 + m^2\right) G(x, x') = 4\pi\delta^{(4)}(x - x')$$

- Green function solution

$$G(x, x') = \theta(s_{\mathrm{t}}) \left[2\delta(s^2) - \frac{m}{s} J_1(ms)\theta(s^2)\right]$$

where $s^\mu = x^\mu - x'^\mu$ and $s = \sqrt{s \cdot s}$

- field equation solution

$$\begin{aligned}
\phi(x) &= \phi_{\mathrm{ext}}(x) - \int d^4x' G_{\mathrm{s}}(x, x') j(x') \\
A^\mu(x) &= A^\mu_{\mathrm{ext}}(x) + \int d^4x' G_{\mathrm{v}}(x, x') J^\mu(x')
\end{aligned}$$

SELF-FIELDS (POINT PARTICLE)

- fields infinite along particle world lines

- define field as average around world line

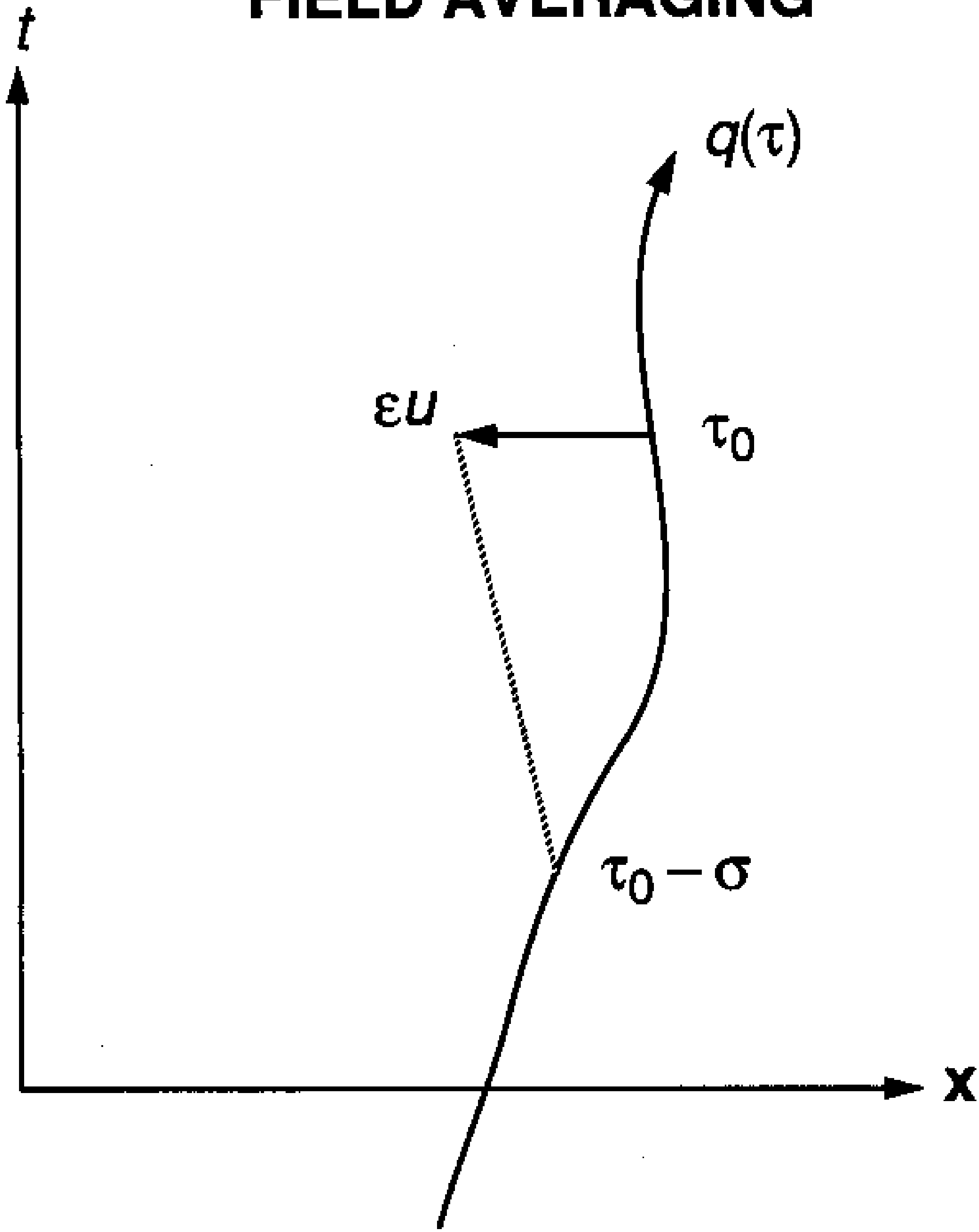
$$\left[\begin{array}{c}\phi(q_i)\\ A^\mu(q_i)\end{array}\right] \equiv \lim_{\epsilon\to 0}\frac{1}{4\pi\epsilon^2}\int_{\Sigma(\epsilon)} d^2\left[\begin{array}{c}\phi(q_i+\epsilon u)\\ A^\mu(q_i+\epsilon u)\end{array}\right]$$

  where $u\cdot u=-1$, $u\cdot\dot{q}_i=0$

- mass renormalization

$$M = M_0 + \frac{2}{3}\left(g_{\rm s}^2 - g_{\rm v}^2\right)\frac{1}{\epsilon} + \frac{1}{2}g_{\rm s}^2 m_{\rm s} - \frac{1}{2}g_{\rm v}^2 m_{\rm v}$$

FIELD AVERAGING



EQUATIONS OF MOTION (POINT PARTICLE)

$$M^* a^\mu = f_{\rm s}^\mu + f_{\rm v}^\mu + g_{\rm s}\mathcal{P}^{\mu\nu}\partial_\nu\phi_{\rm ext} + g_{\rm v}F_{\rm ext}^{\mu\nu}v_\nu$$

where

$$\begin{aligned}
M^* &= \bar{M}^* + g_{\rm s}^2 m_{\rm s}\left[\int_{-\infty}^{\tau} d\tau' \frac{J_1(m_{\rm s}s)}{s} - 1\right] + g_{\rm s}\phi_{\rm ext}(q) \\
\bar{M}^* &= M_0 + g_{\rm s}^2 m_{\rm s} - \frac{2}{3}\left(g_{\rm s}^2 - g_{\rm v}^2\right)\frac{1}{\epsilon} \\
f_{\rm s}^\mu &= \frac{1}{3}g_{\rm s}^2\left(\dot{a}^\mu + a^2 v^\mu\right) - g_{\rm s}^2 m_{\rm s}^2 \mathcal{P}^{\mu\nu}\int_{-\infty}^{\tau} d\tau' \frac{s_\nu}{s^2} J_2(m_{\rm s}s) \\
f_{\rm v}^\mu &= \frac{2}{3}g_{\rm v}^2\left(\dot{a}^\mu + a^2 v^\mu\right) + g_{\rm v}^2 m_{\rm v}^2 \int_{-\infty}^{\tau} d\tau' \frac{s^{[\mu}\dot{q}'^{\nu]}v_\nu}{s^2} J_2(m_{\rm v}s)
\end{aligned}$$

and $s^\mu = q^\mu - q'^\mu$

RELATISTIVISTIC RIGID BODIES

- must specify charge distribution in the particle's rest frame

- use Fermi-Walker coordinates (noninertial) $\xi = (\xi^0, \vec{\xi})$:

  - time coordinate: ξ^0 = "proper time of particle"

  - space coordinate: $\vec{\xi}$ = "position relative to the particle in its rest frame"

  - transport equation: $dA/d\tau = a \wedge v \cdot A$ for any A

  - Jacobian: $\det\left(\frac{\partial x}{\partial \xi}\right) = 1 - a \cdot \xi$

- causality violated for objects larger than $(-a \cdot a)^{-1/2}$

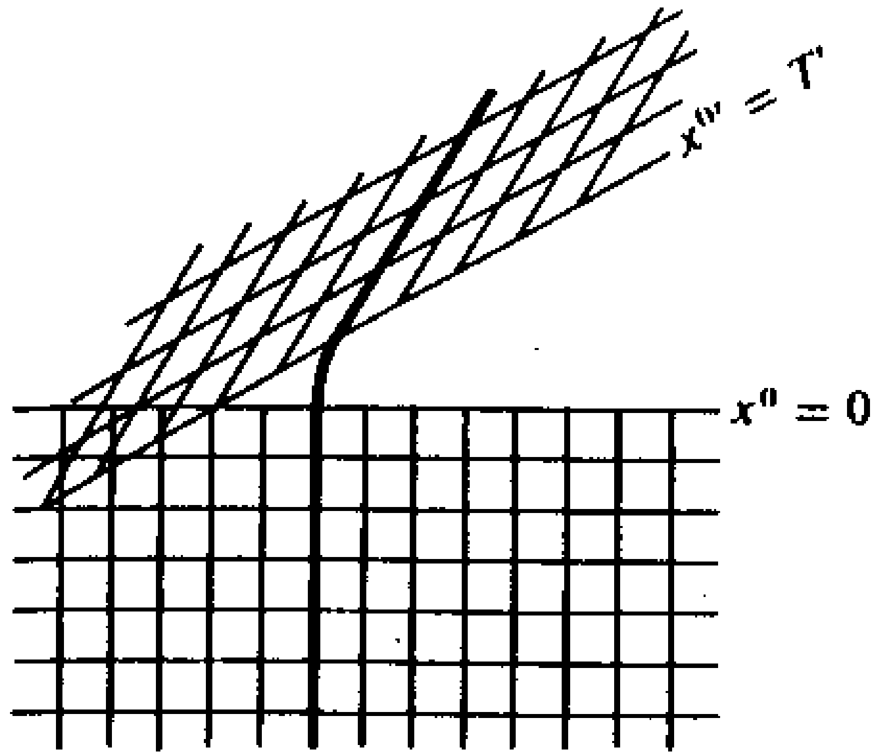
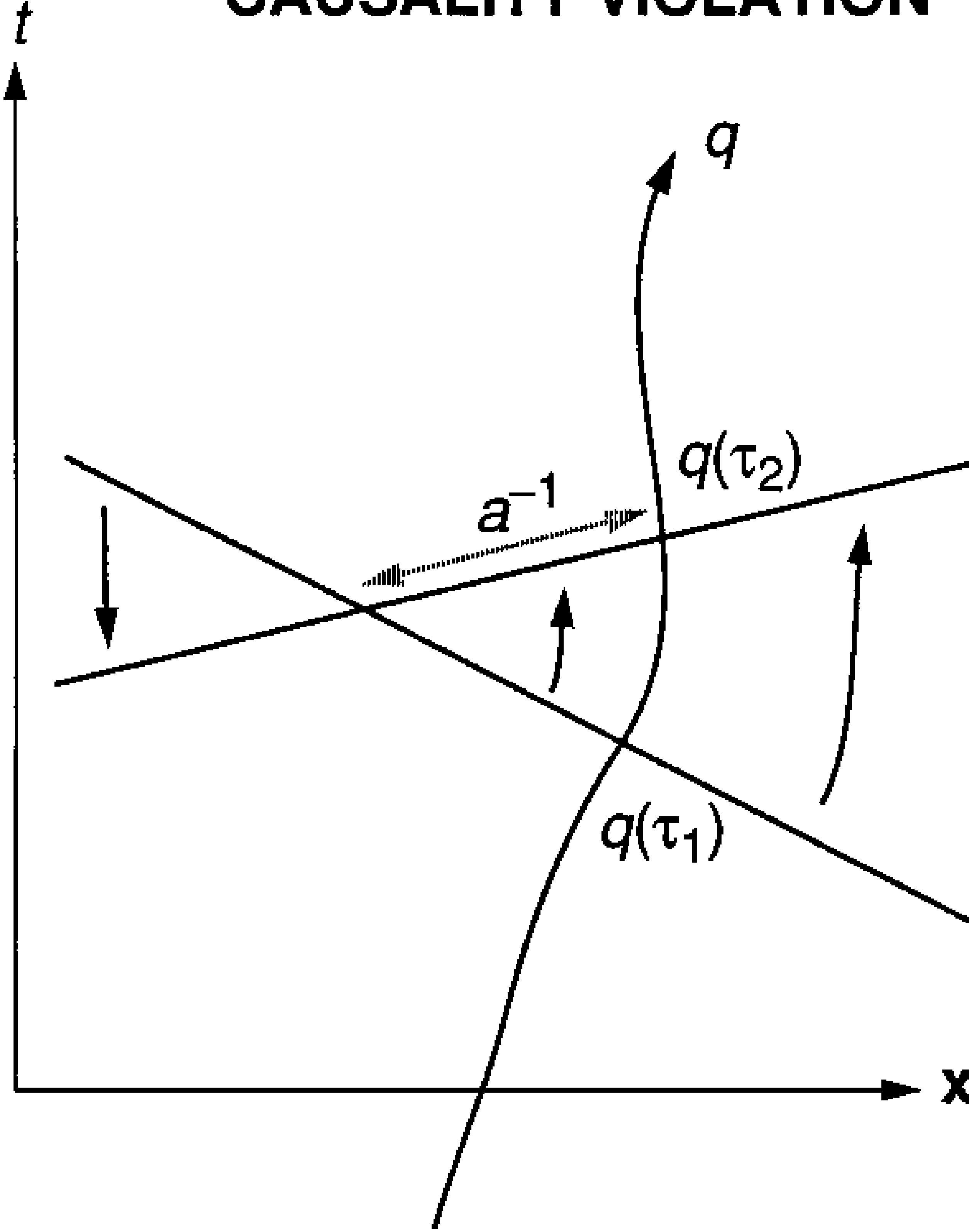


Figure 6.2.

World line of an observer who has undergone a brief period of acceleration. In each phase of motion at constant velocity, an inertial coordinate system can be set up. However, there is no way to reconcile these discordant coordinates in the region of overlap (beginning at distance g^{-1} to the left of the region of acceleration).

from Misner, Thorne, Wheeler, 1973

CAUSALITY VIOLATION



SOURCES FOR EXTENDED NUCLEONS

- sources:

$$
\begin{aligned}
j(x) &= g_\mathrm{S} \sum_i \int d\tau_i \, [1 - a_i \cdot \Delta x_i] \, \delta[v_i \cdot \Delta x_i] \rho \left[\sqrt{-(\Delta x_i)^2} \right] \sqrt{\dot{q}_i^2} \\
J^\mu(x) &= g_\mathrm{V} \sum_i \int d\tau_i \, [1 - a_i \cdot \Delta x_i] \, \delta[v_i \cdot \Delta x_i] \rho \left[\sqrt{-(\Delta x_i)^2} \right] \dot{q}_i^\mu
\end{aligned}
$$

where $\Delta x_i = x - q_i$

- interaction:

$$
\begin{aligned}
-\int d^4x (j\phi + J \cdot A) &= -g_\mathrm{S} \sum_i \int d\tau_i d^3\vec{\xi}_i \, (1 - a_i \cdot \xi_i) \rho(|\vec{\xi}_i|) \phi(x) \sqrt{\dot{q}_i^2} \\
&\quad - g_\mathrm{V} \sum_i \int d\tau_i d^3\vec{\xi}_i \, (1 - a_i \cdot \xi_i) \rho(|\vec{\xi}_i|) A(x) \cdot \dot{q}_i
\end{aligned}
$$

where $x = q_i(\tau_i) + \sum_{k=1}^3 \xi_i^{(k)} e_k(\tau_i)$

DERIVING EQUATIONS OF MOTION

- rewrite Green function integral:

$$A^\mu(x) = \int d^4x' G(x,x') J^\mu(x') = \int d^3\vec{x}' \frac{e^{-R\hat{D}}}{R} J^\mu(\vec{x}', t)$$

  where $R = |\vec{x} - \vec{x}'|$ and $\hat{D} = \sqrt{m^2 + \partial_t^2}$

- evaluate integrals over charge distribution $\rho(\vec{x})$, using nonrelativistic approximation

- use Laplace transform methods to reexpress $\hat{D}$:

$$\int_0^\infty h(s) e^{-2s\hat{D}} = \int_0^\infty ds \left[h\left(\frac{s}{2}\right) - m \int_0^s du\, h\left(\frac{\sqrt{s^2 - u^2}}{2}\right) J_1(mu) \right] \frac{e^{-s\partial_t}}{2}$$

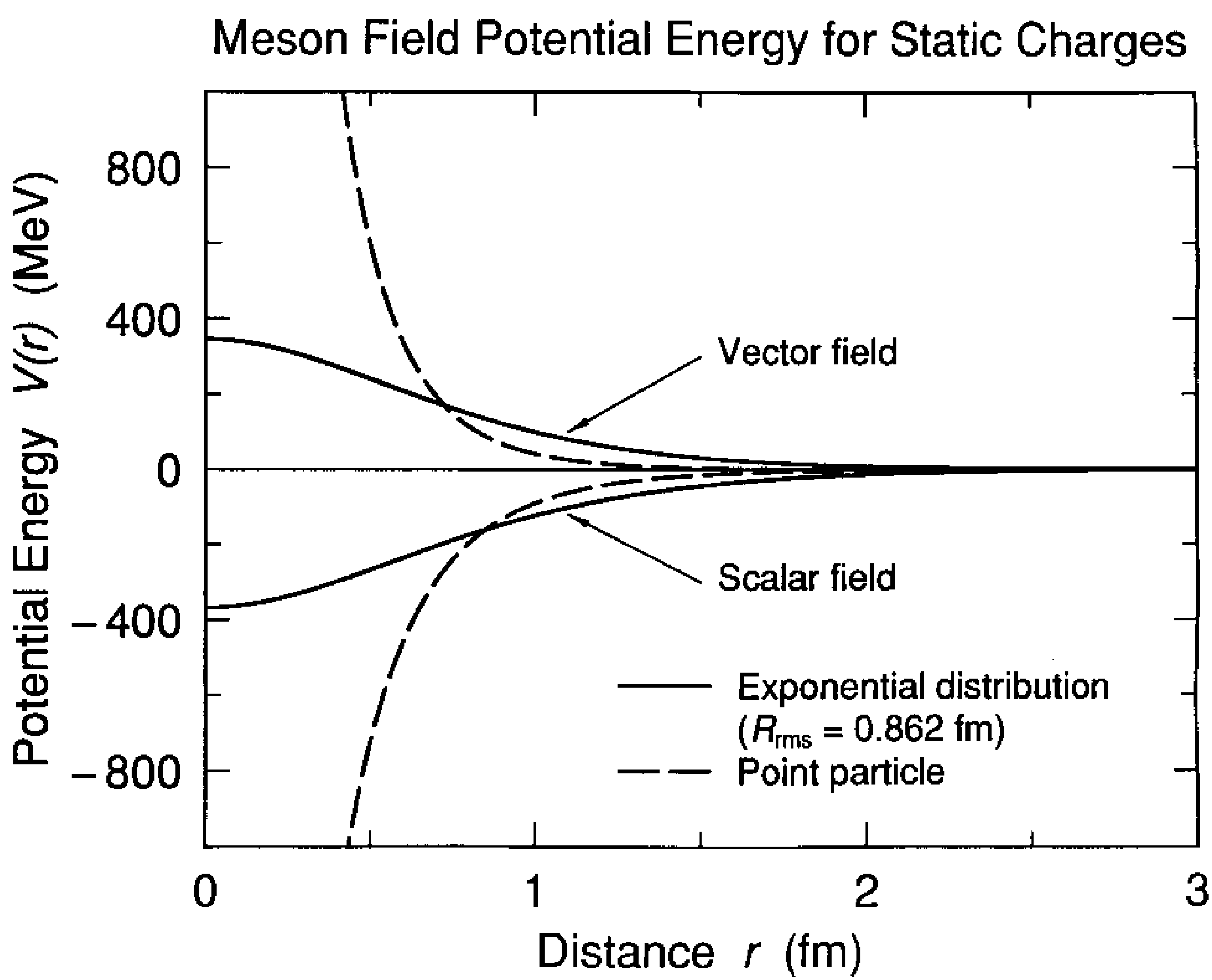
EQUATIONS OF MOTION (EXTENDED PARTICLE)

- equations of motion known exactly for two limiting cases
  - relativistic point particle
  - nonrelativistic extended particle

- use simplest covariant equations generalizing the above:

$$M^* a^\mu = f_{\rm s}^\mu + f_{\rm v}^\mu + g_{\rm s} \mathcal{P}^{\mu\nu} \partial_\nu \phi_{\rm ext} + g_{\rm v} F_{\rm ext}^{\mu\nu} v_\nu$$

  where $v = \dot{q}$, $a = \ddot{q}$, $\mathcal{P}^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$



Meson Field Potential Energy for Static Charges
Potential Energy V(r) (MeV)
800
400
0
−400
−800
Vector field
Scalar field
Exponential distribution
(R_{rms} = 0.862 fm)
Point particle
0
1
2
3
Distance r (fm)

EFFECTIVE MASS

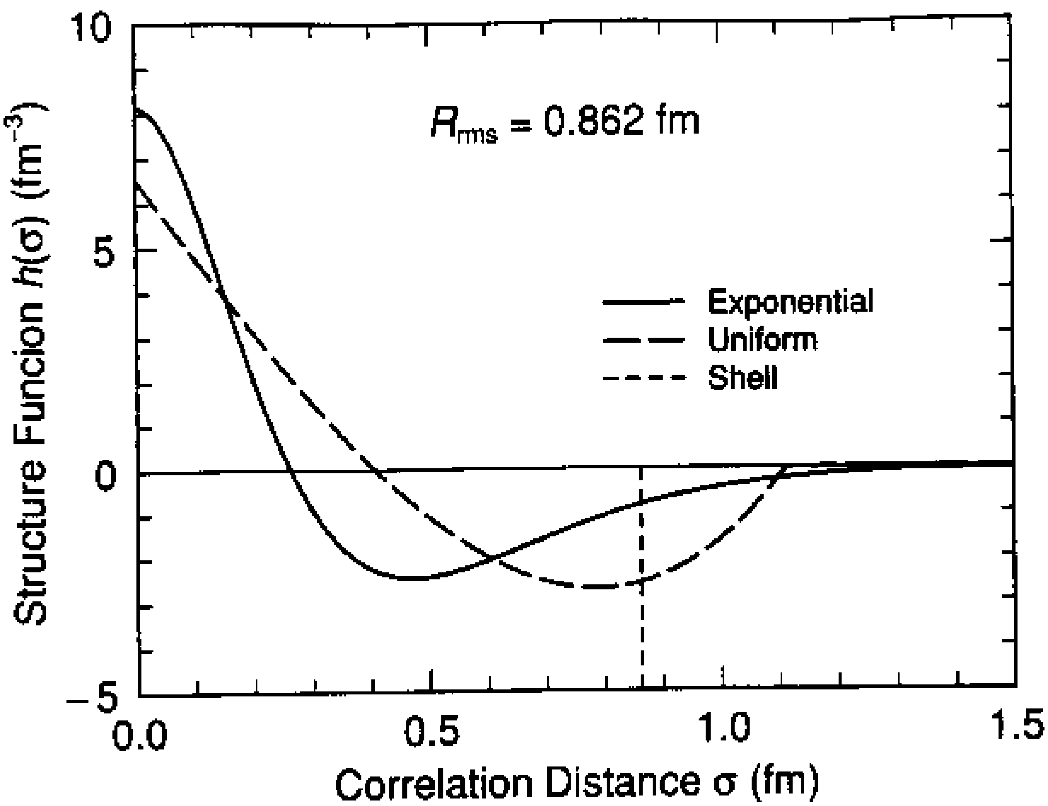
- effective mass:

$$
\begin{aligned}
M^* &= \tilde{M}^* + \Delta M_{\text{self}} + g_{\text{s}}\phi_{\text{ext}} \\
\tilde{M}^* &= M - \frac{g_{\text{s}}^2}{6}\left[\frac{H_0(m_{\text{s}})}{m_{\text{s}}} - 2H_1(m_{\text{s}})\right] - \frac{g_{\text{v}}^2}{6}\left[5\frac{H_0(m_{\text{v}})}{m_{\text{v}}} + 2H_1(m_{\text{v}})\right]
\end{aligned}
$$

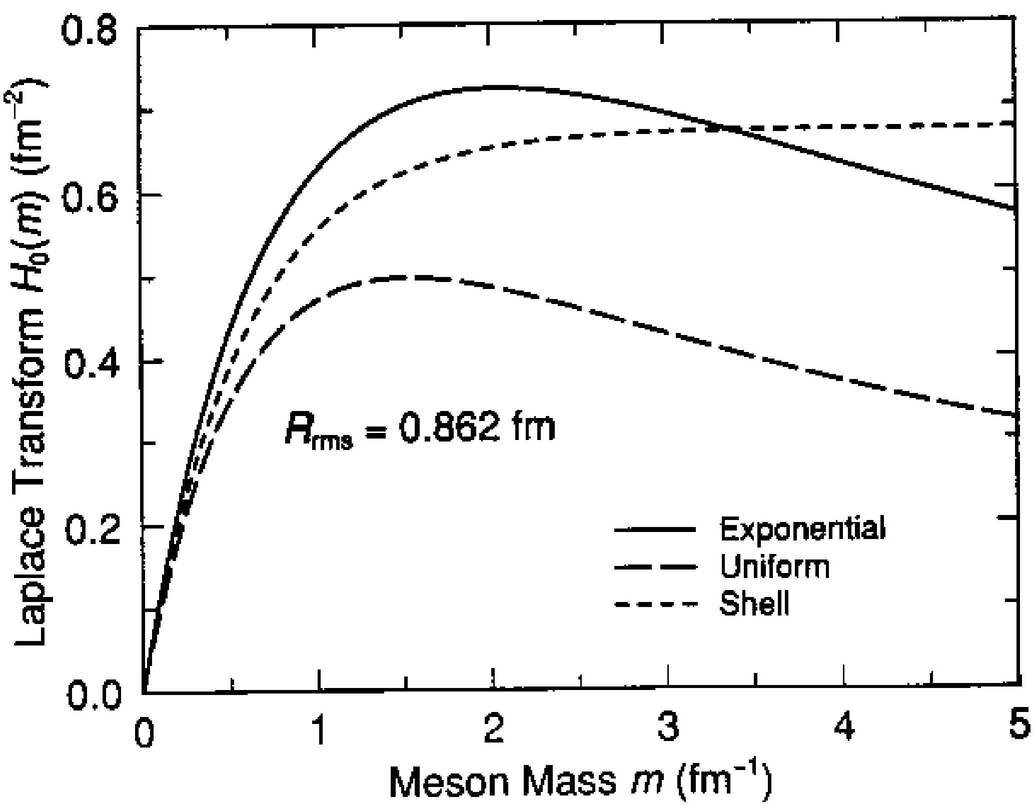
- nucleon structure functions:

$$
\begin{aligned}
H_n(m) &= \int_0^\infty ds\, h(s) s^n e^{-2ms} \\
h(s) &= 32\pi^2 \int_0^\infty ds' \left(s'^{\,2} - s^2\right) \rho(s+s')\rho(|s-s'|) \\
h'(s) &= dh/ds \\
\bar{h}(s) &= \int_0^s h \\
\tilde{h}(s) &= h'(s) - 6m_{\text{v}}^2 \bar{h}(s)
\end{aligned}
$$

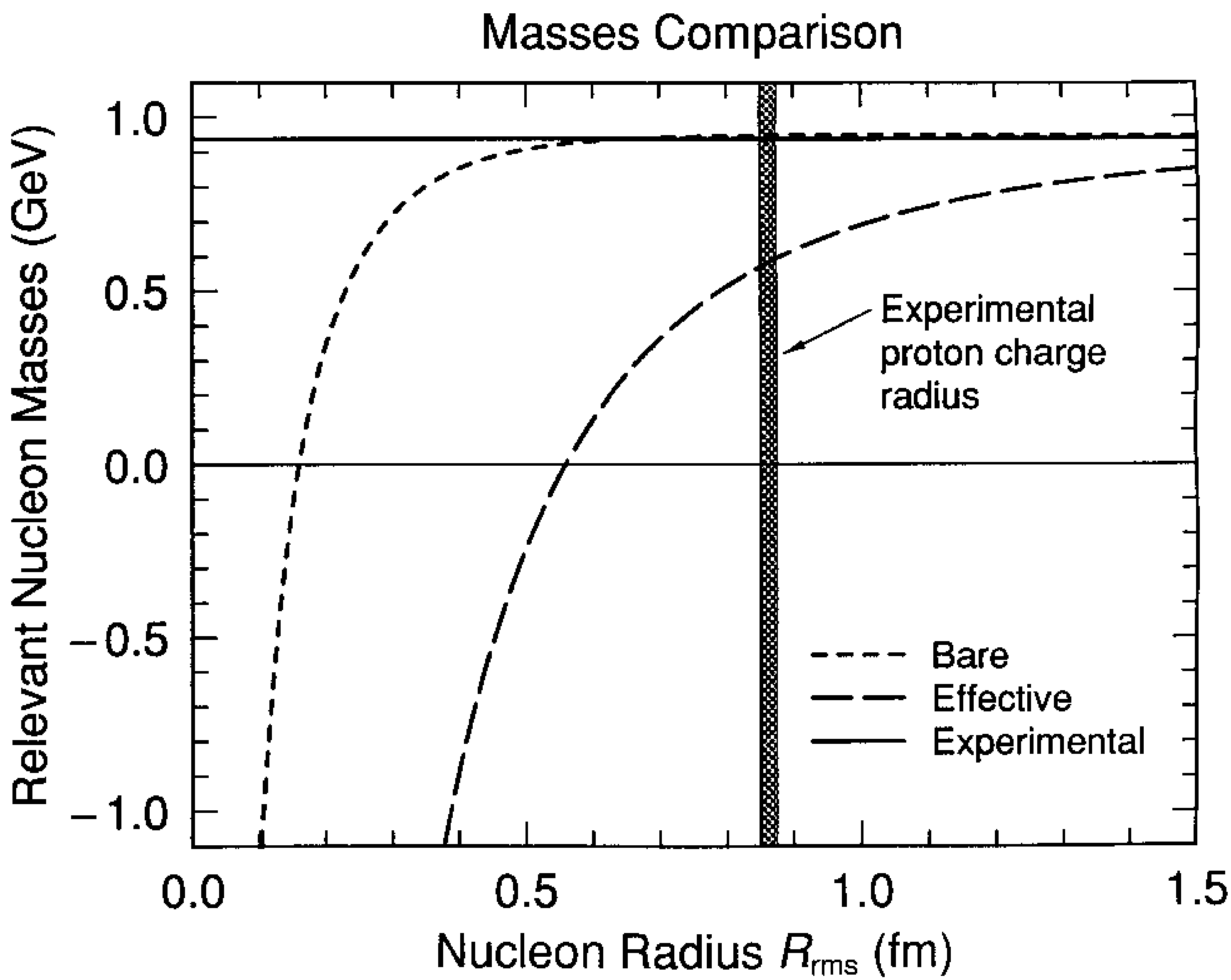
where $\rho(r)$ is the nucleon mass density



R_{rms} = 0.862 fm
Exponential
Uniform
Shell
Structure Funcion $h(\sigma)$ (fm^{-3})
Correlation Distance σ (fm)
10
5
0
−5
0.0
0.5
1.0
1.5



R_{rms} = 0.862 fm
Exponential
Uniform
Shell
Laplace Transform $H_0(m)$ (fm^{-2})
Meson Mass m (fm^{-1})
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5


Masses Comparison
Relevant Nucleon Masses (GeV)
1.0
0.5
0.0
−0.5
−1.0
0.0
0.5
1.0
1.5
Nucleon Radius R_{rms} (fm)
Experimental
proton charge
radius
Bare
Effective
Experimental

SELF-INTERACTION (SCALAR)

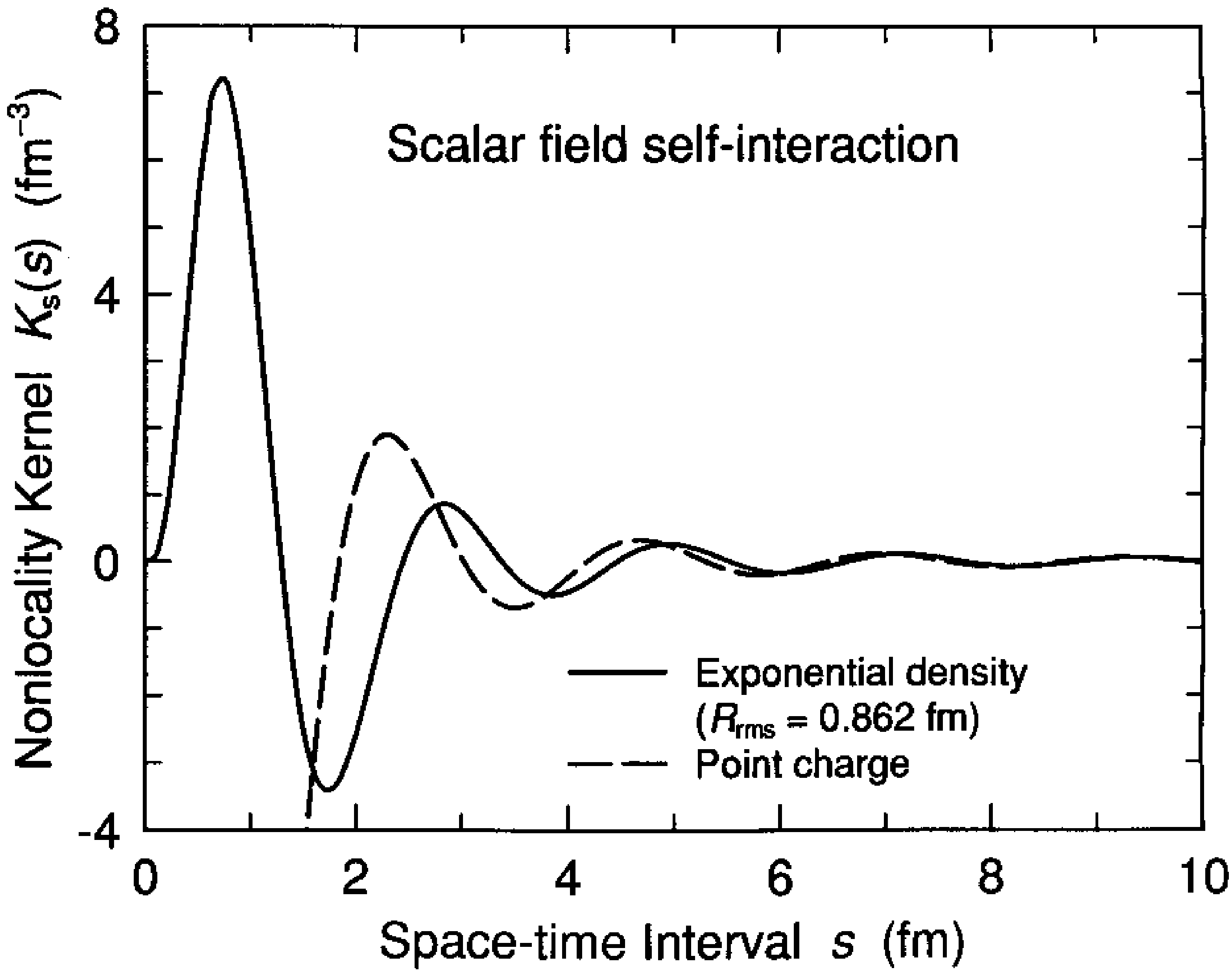
- scalar self-mass:

$$\Delta M_{\rm self} = -g_{\rm s}^2 \int_0^\infty d\sigma \left[\bar{h}\left(\frac{\sigma}{2}\right) - m_{\rm s} \int_0^s du\, \bar{h}\left(\frac{\sqrt{s^2-u^2}}{2}\right) J_1(m_{\rm s}u) \right] + g_{\rm s}^2 \frac{H_0(m_{\rm s})}{m_{\rm s}}$$

- scalar self-force:

$$f_{\rm s}^\mu = \frac{g_{\rm s}^2}{12} \mathcal{P}^{\mu\nu} \int_0^\infty d\sigma \left[h'\left(\frac{\sigma}{2}\right) - m_{\rm s} \int_0^s du\, h'\left(\frac{\sqrt{s^2-u^2}}{2}\right) J_1(m_{\rm s}u) \right] s_\nu$$

where $s^\mu = q^\mu(\tau) - q^\mu(\tau - \sigma)$ and $s = \sqrt{s \cdot s}$



Scalar field self-interaction
Nonlocality Kernel $K_s(s)$ (fm^{-3})
Space-time Interval s (fm)
Exponential density
(R_{rms} = 0.862 fm)
Point charge
8
4
0
-4
0
2
4
6
8
10

SELF-INTERACTION (VECTOR)

- vector self-force:

$$f^\mu_{\rm V} = \frac{g^2_{\rm V}}{6}\mathcal{P}^{\mu\nu}\int_0^\infty d\sigma\left\{\tilde{h}\left(\frac{\sigma}{2}\right) - m_{\rm V}\int_0^s d\zeta\left[\mathcal{W}\tilde{h}\left(\frac{\sqrt{s^2-\zeta^2}}{2}\right)\right.\right.$$

$$\left.\left. + \left(\mathcal{W} - v'\cdot v\right) h'\left(\frac{\sqrt{s^2-\zeta^2}}{2}\right)\right] J_1(m_{\rm V}\zeta)\right\} s_\nu$$

where $s^\mu = q^\mu(\tau) - q^\mu(\tau-\sigma)$, $s = \sqrt{s\cdot s}$ and

$$\mathcal{W} = \frac{(s\cdot v')(s\cdot v)}{s^2}$$

PREACCELERATION

- assume exponential growth of acceleration:

$$a^i(\tau) \sim C^i e^{\kappa\tau}$$

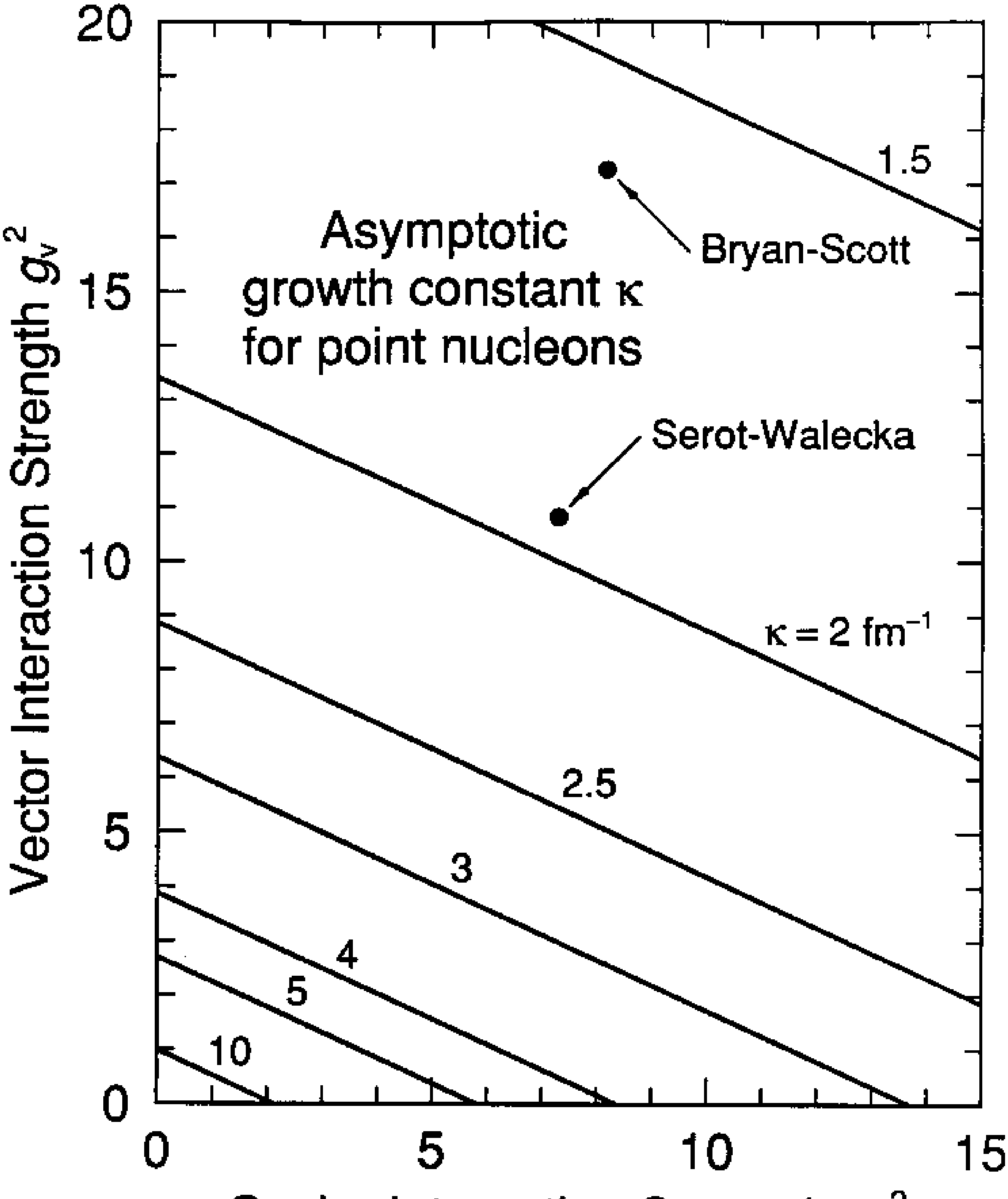
- characteristic equation:

$$\frac{g_{\rm s}^2}{3}\left[D_{\rm s}H_0(D_{\rm s}) - m_{\rm s}H_0(m_{\rm s})\right] + \frac{g_{\rm v}^2}{3}\left[\frac{3\kappa^2 - {D_{\rm v}}^2}{D_{\rm v}}H_0(D_{\rm v}) + m_{\rm v}H_0(m_{\rm v})\right]$$

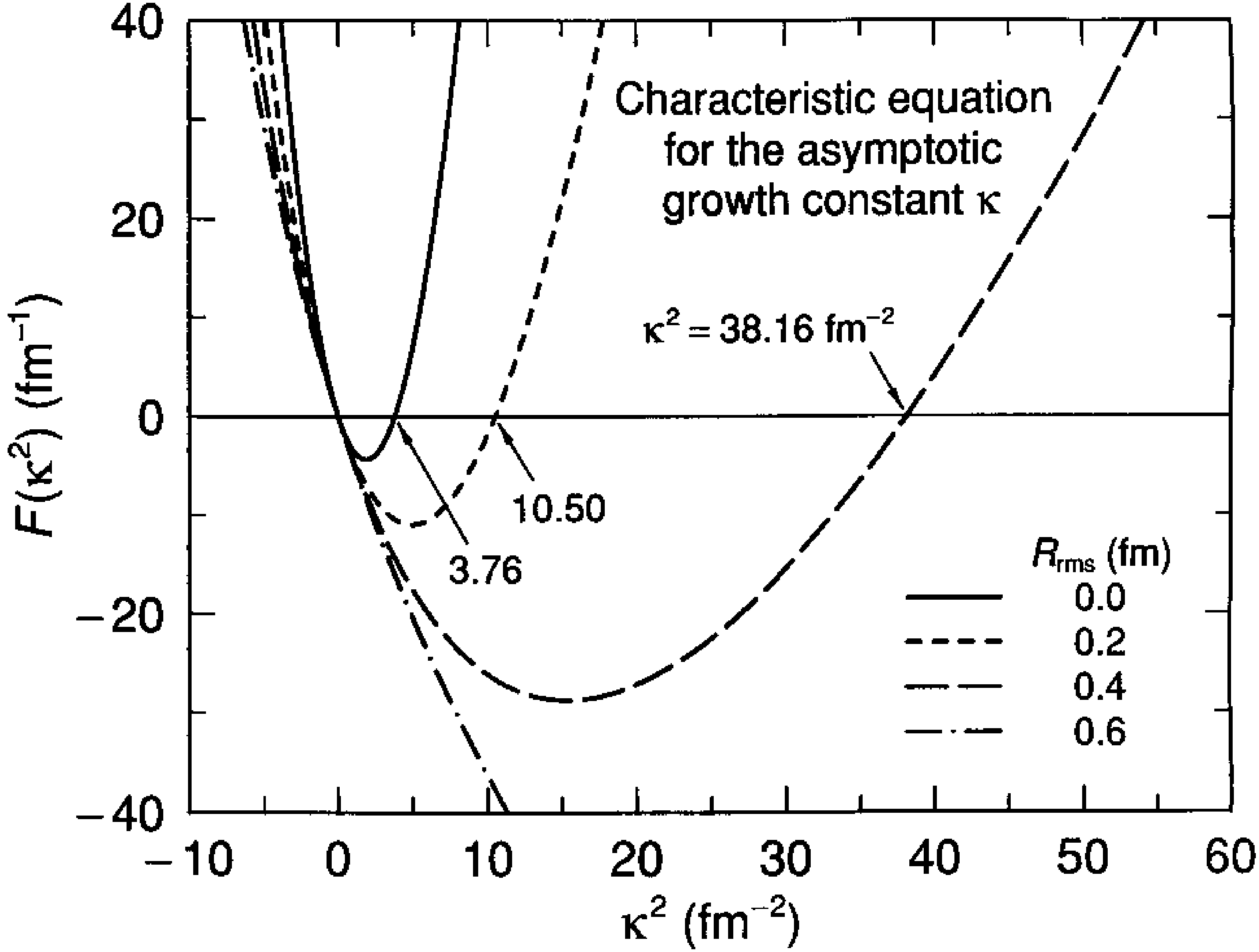
$$+\tilde{M}^*\kappa^2 = 0$$

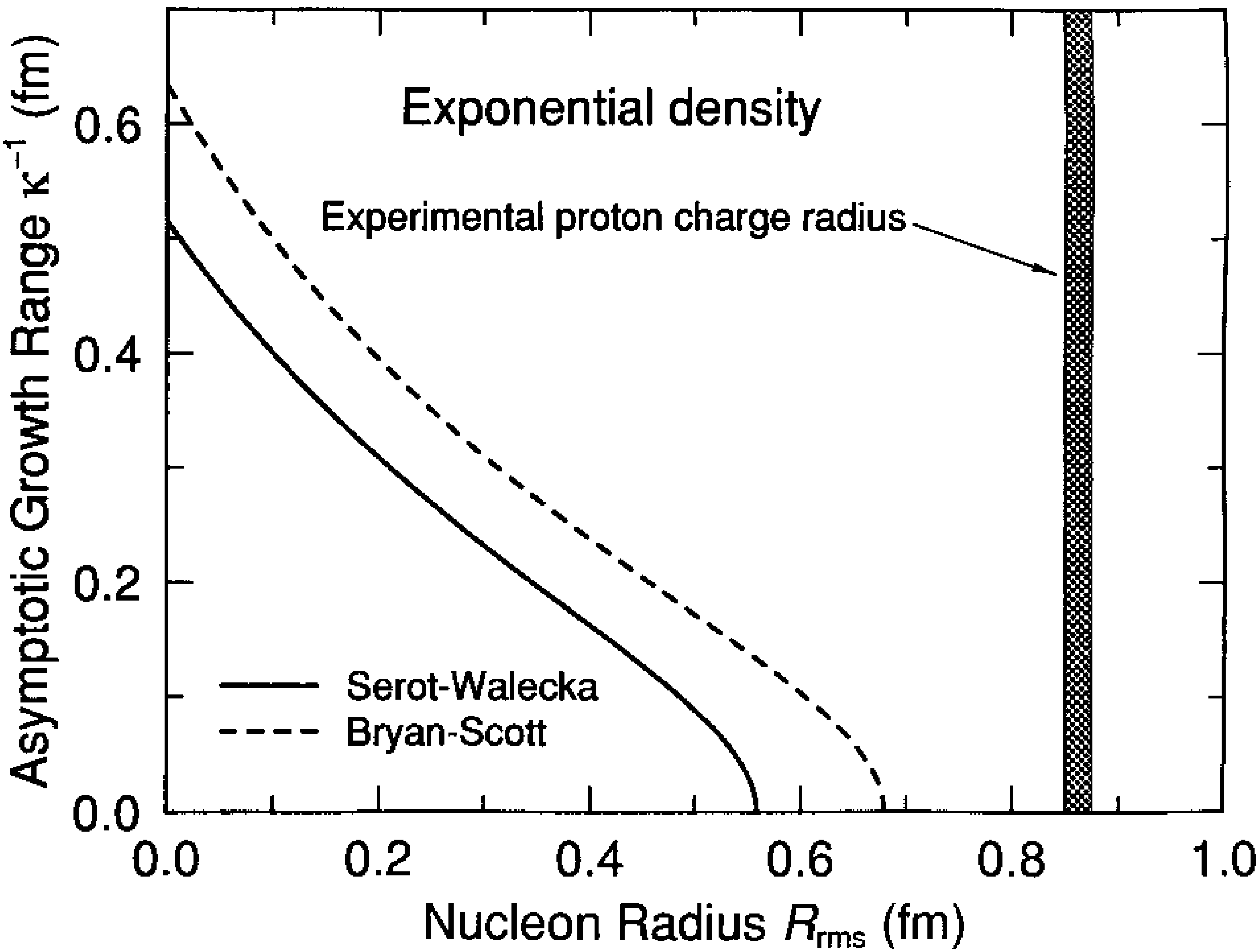
where $D_{\rm v,s} = \sqrt{{m_{\rm v,s}}^2 + \kappa^2}$

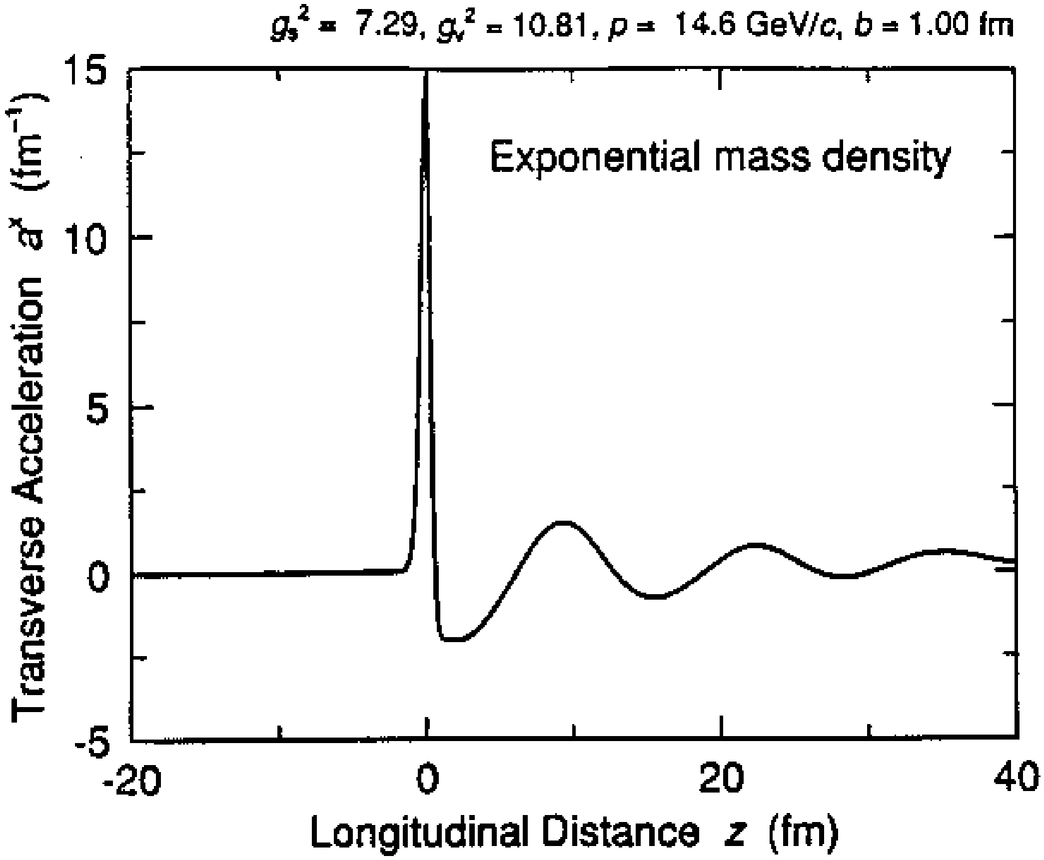
- preacceleration occurs when a solution κ has real positive part

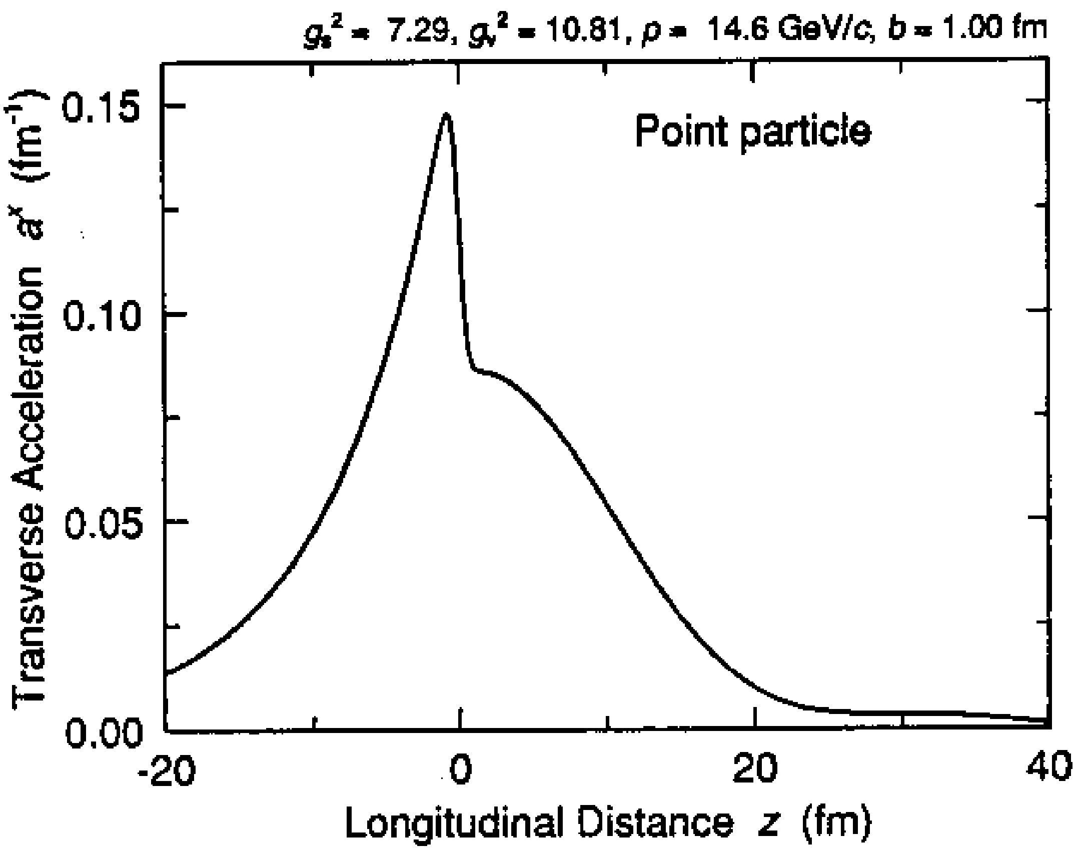


Asymptotic
growth constant κ
for point nucleons
Bryan-Scott
Serot-Walecka
1.5
κ = 2 fm^{-1}
2.5
3
4
5
10
Vector Interaction Strength g_v^2
Scalar Interaction Strength g_s^2
0
5
10
15
20


Characteristic equation
for the asymptotic
growth constant κ
$\kappa^2 = 38.16\ \mathrm{fm}^{-2}$
10.50
3.76
R_{rms} (fm)
0.0
0.2
0.4
0.6
$F(\kappa^2)$ (fm^{-1})
κ^2 (fm^{-2})
40
20
0
−20
−40
−10
0
10
20
30
40
50
60


Exponential density
Experimental proton charge radius
Serot-Walecka
Bryan-Scott
Asymptotic Growth Range κ⁻¹ (fm)
Nucleon Radius R_{rms} (fm)
0.0
0.2
0.4
0.6
0.8
1.0

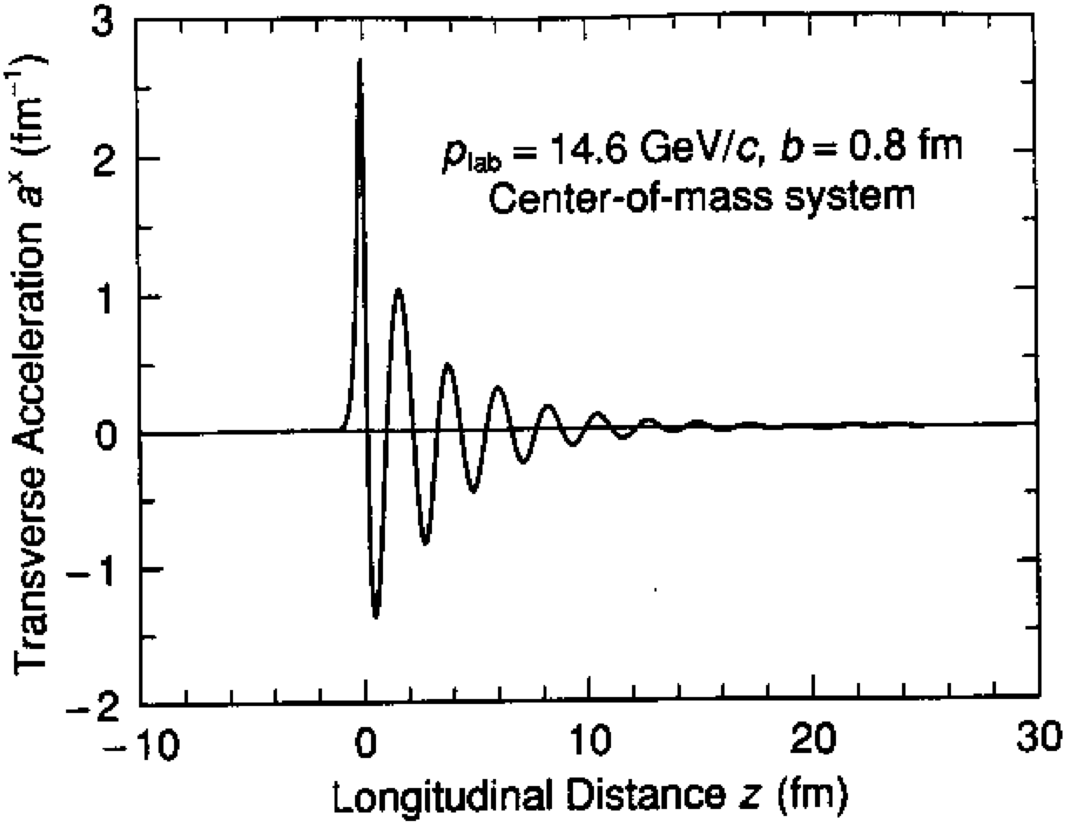

$g_s^2 = 7.29$, $g_v^2 = 10.81$, $p = 14.6$ GeV/c, $b = 1.00$ fm
Exponential mass density
Transverse Acceleration a^x (fm^{-1})
Longitudinal Distance z (fm)
15
10
5
0
-5
-20
0
20
40


$g_s^2 = 7.29$, $g_v^2 = 10.81$, $p = 14.6$ GeV/c, $b = 1.00$ fm
Point particle
Transverse Acceleration a^x (fm^{-1})
Longitudinal Distance z (fm)
0.15
0.10
0.05
0.00
-20
0
20
40

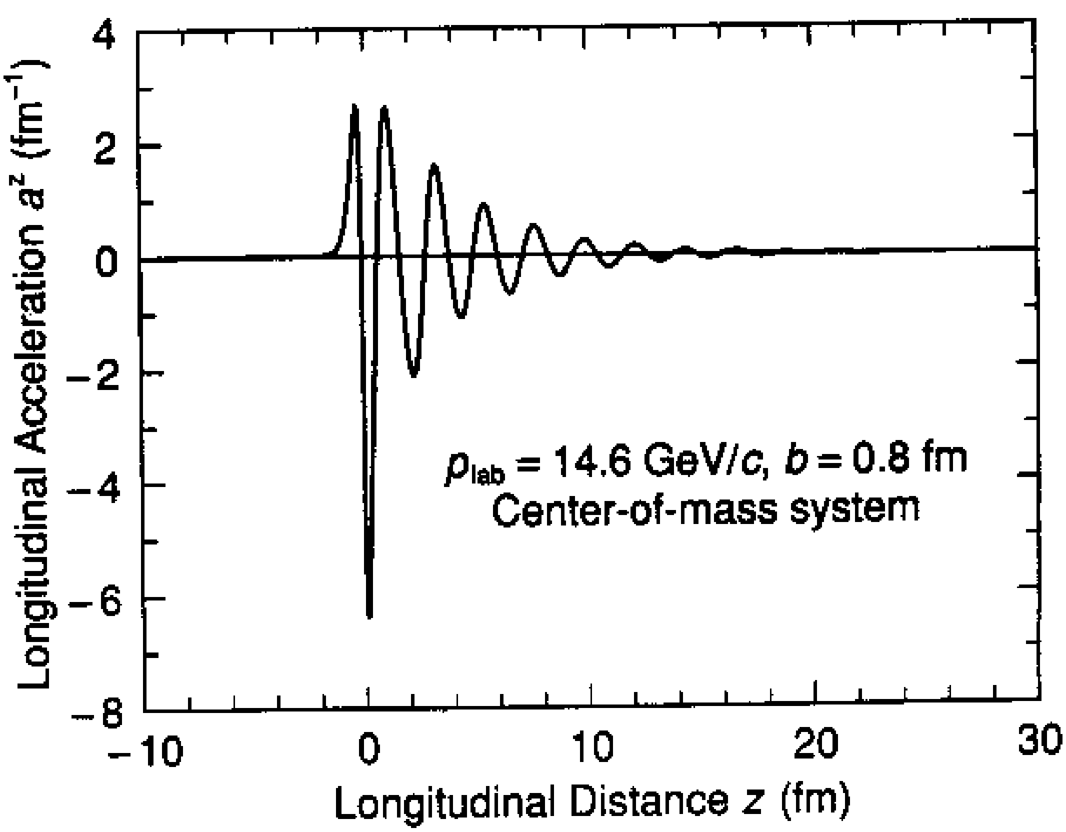
LENGTH SCALE COMPARISON

quantity	value*	
	ED	*HD*
Compton radius, M^{-1}	386.15 fm	0.21 fm
Potential range, m^{-1}	∞	0.36 fm (s)
		0.25 fm (v)
Classical radius, κ^{-1}	2.81 fm	0.52 fm
Experimental radius, r	0	0.86 fm

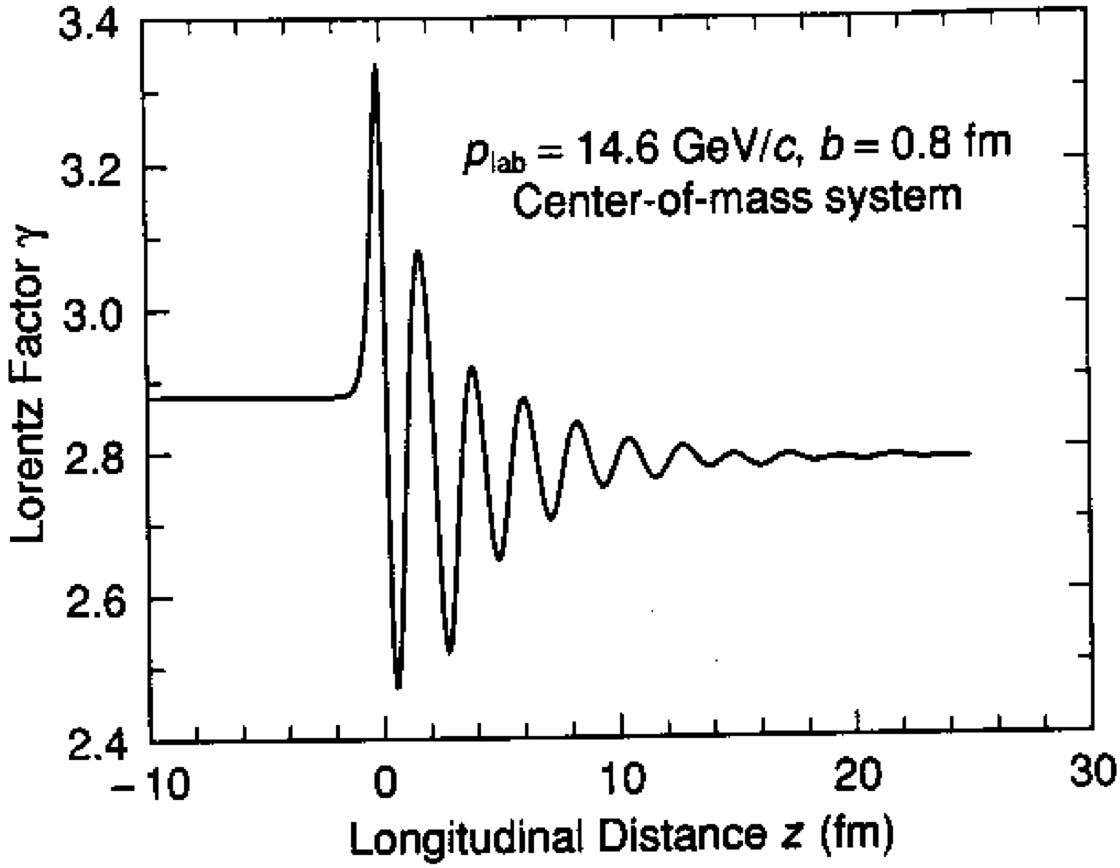
*ED = electrodynamics, HD = hadrodynamics, s = scalar, v = vector



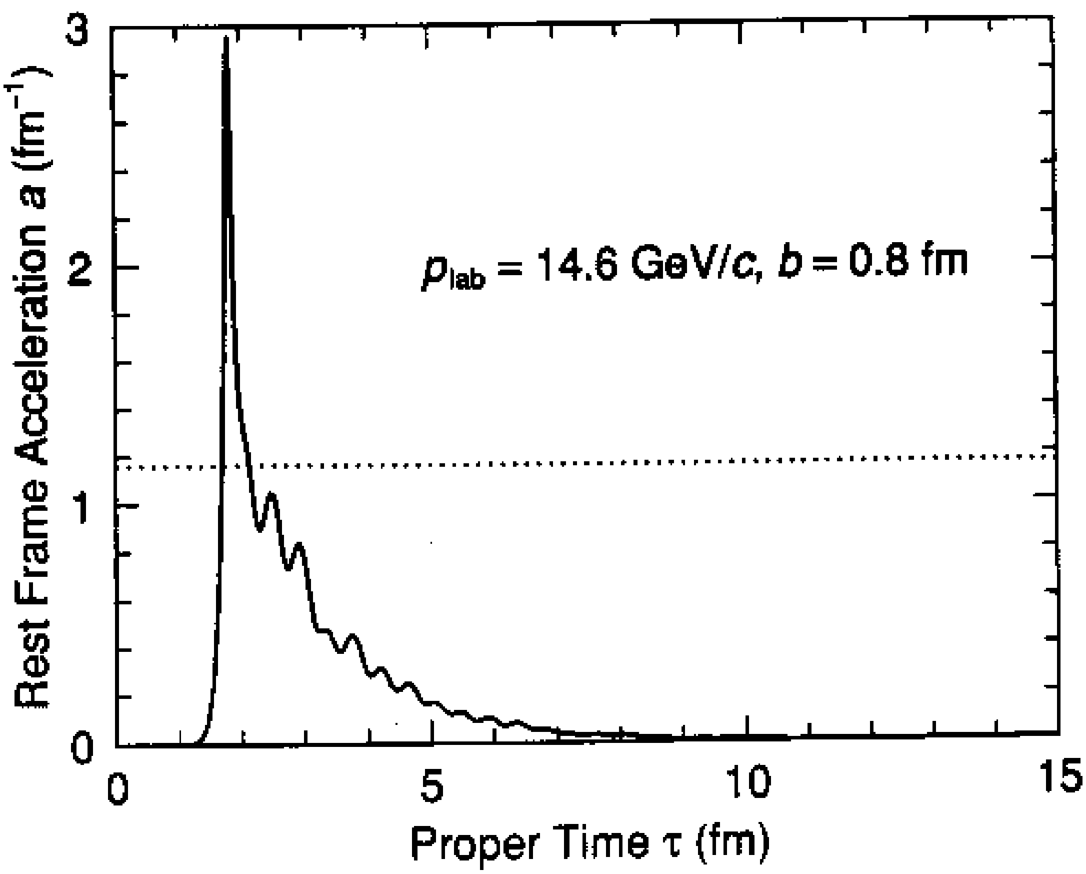
$p_{lab} = 14.6$ GeV/c, $b = 0.8$ fm
Center-of-mass system
Transverse Acceleration a^x (fm^{-1})
Longitudinal Distance z (fm)
3
2
1
0
−1
−2
−10
0
10
20
30



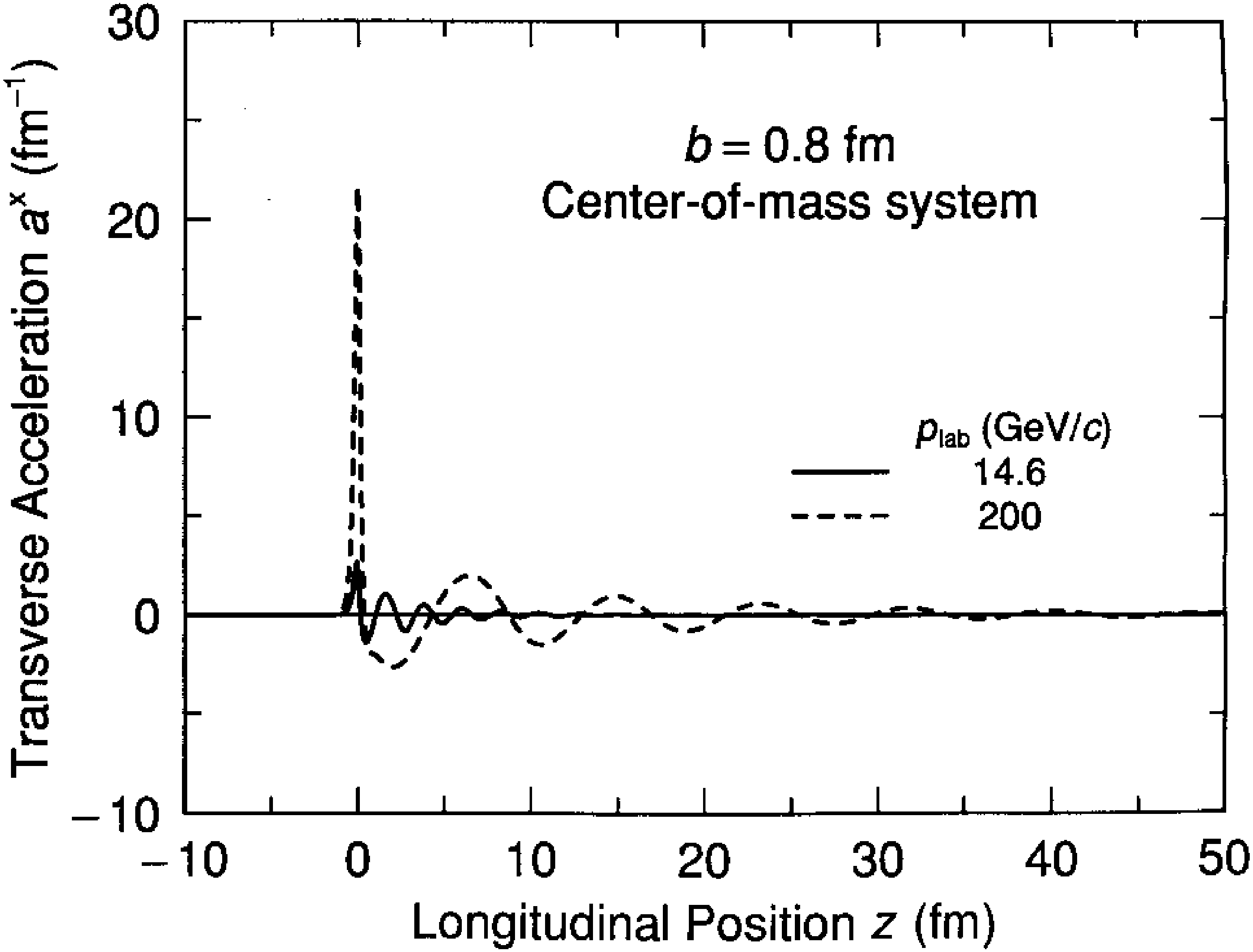
$p_{lab} = 14.6$ GeV/c, $b = 0.8$ fm
Center-of-mass system
Longitudinal Acceleration a^z (fm^{-1})
Longitudinal Distance z (fm)
4
2
0
−2
−4
−6
−8
−10
0
10
20
30



p_{lab} = 14.6 GeV/c, b = 0.8 fm
Center-of-mass system
Lorentz Factor γ
Longitudinal Distance z (fm)
3.4
3.2
3.0
2.8
2.6
2.4
−10
0
10
20
30



p_{lab} = 14.6 GeV/c, b = 0.8 fm
Rest Frame Acceleration a (fm^{-1})
Proper Time τ (fm)
3
2
1
0
0
5
10
15



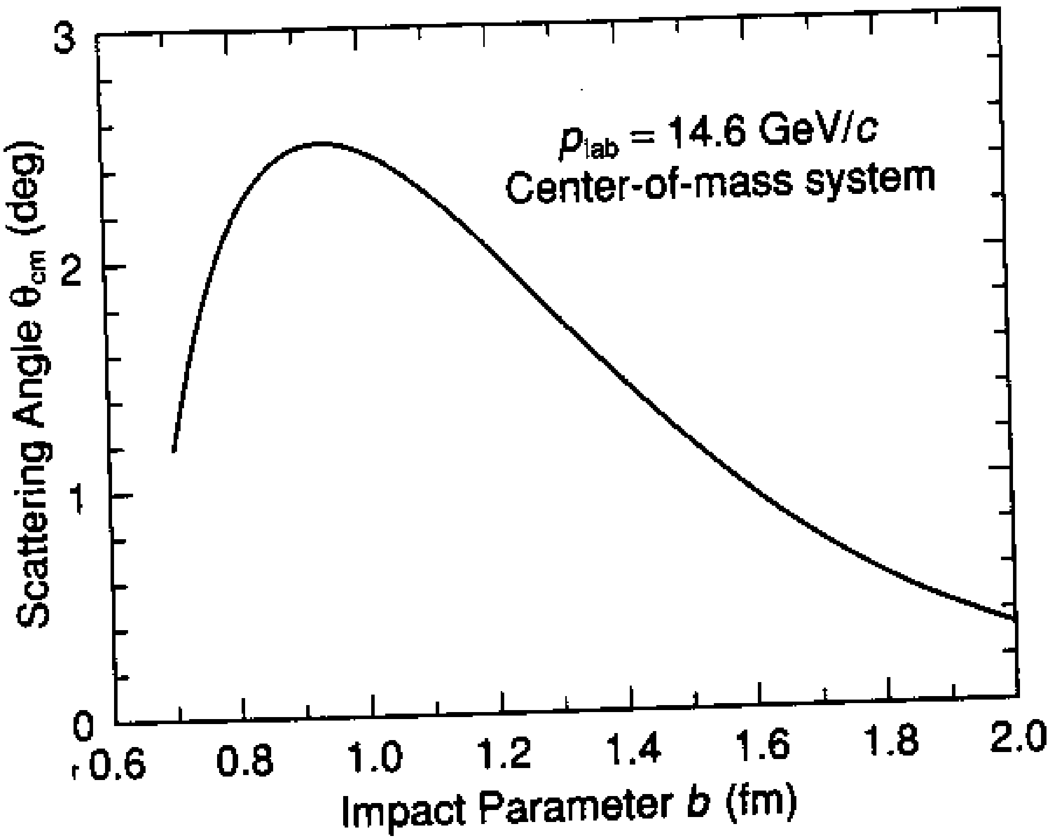
30
20
10
0
−10
−10
0
10
20
30
40
50
Transverse Acceleration a^x (fm^{-1})
Longitudinal Position z (fm)
$b = 0.8$ fm
Center-of-mass system
p_{lab} (GeV/c)
14.6
200

CROSS-SECTION

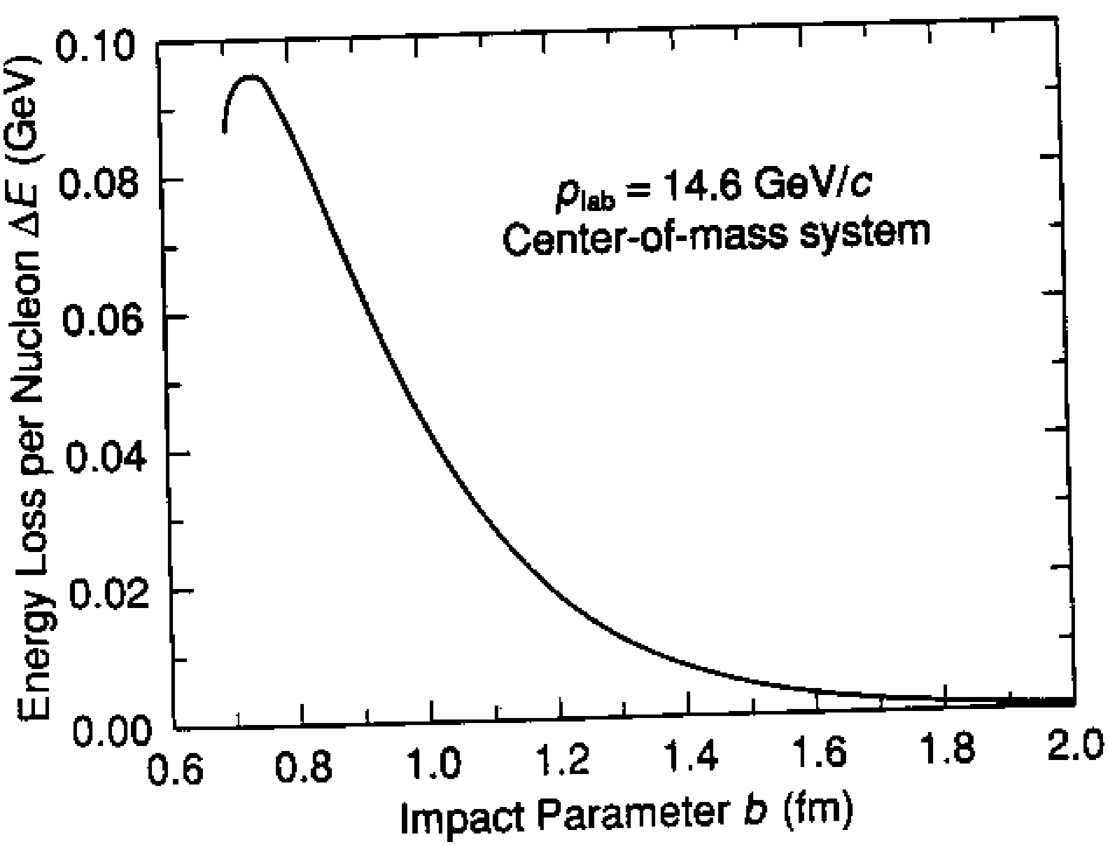
- classical cross-section obtained from relationship between impact parameter b and scattering angle θ

- $\frac{d\sigma}{d\Omega} = \sum_i \frac{b_i}{\sin\theta_i} \left|\frac{db}{d\theta}\right|_i$

- $\theta = \theta(b)$ may have a maximum, which implies $b = b(\theta)$ is double-valued; this is called "rainbow scattering", and is a common feature of classical scattering



p_{lab} = 14.6 GeV/c
Center-of-mass system
Scattering Angle θ_{cm} (deg)
0
1
2
3
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Impact Parameter b (fm)



p_{lab} = 14.6 GeV/c
Center-of-mass system
Energy Loss per Nucleon ΔE (GeV)
0.00
0.02
0.04
0.06
0.08
0.10
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Impact Parameter b (fm)

HISTORICAL PERSPECTIVE (I)

- P. A. M. Dirac (1939): Lorentz-Dirac equation

- H. J. Bhabha (1939): equations of motion for vector mesons

- H. J. Bhabha and Harish-Chandra (1944): equations of motion for tensor mesons

HISTORICAL PERSPECTIVE (II)

- P. Havas (1952): alternative versions of equations of motion for vector and scalar mesons

- E. Moniz and D. Sharp (1977): quantum-mechanical analysis of self-interaction for nonrelativistic electrodynamics

- L. Wilets *et al.* (1977): classical, nonrelativistic model for heavy-ion collisions using two-body force

COMPARISON WITH BUU APPROACH

comparison	*BUU**	*CRHD**
field	classical mean field	exact classical field
self-interaction	absent	present
collisions	two-body	N-body
quantum effects	initial conditions & collisions	initial conditions
nucleon structure	point	rigid body

*BUU = Boltzmann-Uehling-Uhlenbeck approach, CRHD = Classical Relativistic Hadrodynamics

FUTURE INVESTIGATION

- causality violation due to rigid body assumption
- radiation estimates / meson production
- systematics for two-particle case
- multinucleon calculations
- quantum corrections

CONCLUSION

classical hadrodynamics for extended nucleons

- provides a natural Lorentz-covariant microscopic approach to relativistic nucleus-nucleus collisions
- satisfies *a priori* the basic conditions that are present
- requires minimal physical input
- leads to equations of motion that can be solved exactly
- cures difficulties with preacceleration and runaway solutions
- contains an inherent spacetime nonlocality that may be responsible for significant collective effects